Rational Functions

1] Find any holes
2] Find the vertical asymptote
3]Find any zeros
4] Find the y-int (if any)
5] Find the horizontal or oblique asymptote
6] Sketch the graph
7] Identify the domain and range

Step 1: Look for holes

1] Factor completely and cancel common factors
2] Factors that cancel form holes in the graph. To find the x-coordinate of a hole, set the canceled factor equal to zero and solve for x.
3] Substitute that result back in to the simplified form to find the y-coordinate of the hole.

A] $y = \dfrac{2x^2 - 8}{x^2 - x - 6}$

B] $y = \dfrac{x^2 + x - 12}{x + 3}$

Steps 2 - 4: Find vertical asymptotes, zeros, and y-intercept

1] Set the denominator equal to zero to find any vertical asymptotes
2] Set the numerator equal to zero to find any zeros
3] Substitute 0 for x and solve for y to find the y-intercept, if any

A] $y = \dfrac{2x^2 - 8}{x^2 - x - 6} = \dfrac{2\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-3)}$

B] $y = \dfrac{x^2 + x - 12}{x + 3} = \dfrac{(x+4)(x-3)}{x+3}$

Step 5: Find horizontal OR oblique (SLANT) asymptote (can't have both)

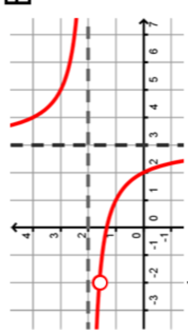
1] If the degree of the numerator is LESS than the degree of the denominator, then y=0 is the horizontal asymptote.
2] If the degree of the numerator EQUALS the degree of the denominator, then the ratio of leading coefficients is the horizontal asymptote.
3] If the degree of the numerator is MORE than the degree of the denominator, then divide to find the asymptote, ignoring the remainder. When the difference in degree is one, the asymptote will be oblique.

A] $y = \dfrac{2x^2 - 8}{x^2 - x - 6} = \dfrac{2\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-3)}$

B] $y = \dfrac{x^2 + x - 12}{x + 3} = \dfrac{(x+4)(x-3)}{x+3}$

Steps 6 - 7: Graph, Domain, and Range

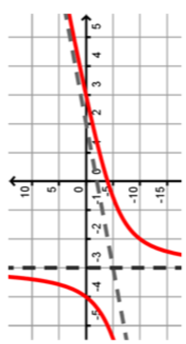
A]



Domain:

Range:

B]



Domain:

Range: