5B.1 - Notes on INVERSES Name:_____

GSE Algebra II Date:_____

What is the INVERSE of a Function?

- It is essentially changing the ordered pair.
- Switching of the x value and y value of a point

Inverses Algebraically – How do we find the Inverse of a function algebraically?

1. Step 1 – change $f(x)$ to a y value
2. Step 2 – switch the x value and the y value in the equation
3. Step 3 – solve for y
4. Step 4 – replace y with $f^{-1}(x)$

Examples

1.) $f(x)=3x-7$

$y=3x-7$

$x=3y-7$

$+7 \quad +7$

$\frac{3y}{3}=\frac{x+7}{3}$

$f^{-1}(x)=\frac{x+7}{3}$

2.) $f(x)=4x^2$

$y=4x^2$

$x=4y^2$

$\frac{x}{4}=\frac{4y^2}{4}$

$\sqrt{y^2}=\sqrt{\frac{x}{4}}$

$f^{-1}(x)=\pm\sqrt{\frac{x}{4}}$

3.) $f(x)=x^3+4$

$y=x^3+4$

$x=y^3+4$

$-4 \quad -4$

$\sqrt[3]{y^3}=\sqrt[3]{x-4}$

$y=\sqrt[3]{x-4}$

$f^{-1}(x)=\sqrt[3]{x-4}$

4.) $f(x)=\sqrt{x-2}+5$

$y=\sqrt{x-2}+5$

$x=\sqrt{y-2}+5$

$-5 \quad -5$

$(\sqrt{y-2})^2=(x-5)^2$

$y-2=(x-5)^2$

$+2 \quad +2$

$f^{-1}(x)=(x-5)^2+2$

5.) $y=3^x$

$x=3^y$

$\log_3 x=\log_3 3^y$

$y=\log_3 x$

$f^{-1}(x)=\log_3 x$

6.) $y=2^{x-1}$

$x=2^{y-1}$

$\log_2 x=\log_2 2^{y-1}$

$y-1=\log_2 x$

$+1 \quad +1$

$y=\log_2(x)+1$

$f^{-1}(x)=1+\log_2 x$

$=\log_2(x)+1$

7.) $y=\log_3 x$

$3^x=3^{\log_3 y}$

$3^x=y$

$f^{-1}(x)=3^x$

8.) $y=\log_4(x+2)$

$4^x=4^{\log_4(y+2)}$

$y+2=4^x$

$-2 \quad -2$

$y=4^x-2$

$f^{-1}(x)=4^x-2$

How to Determine if 2 functions are Inverses of Each other

1. Step 1 – do the composition of each function
   a. f(g(x)) = x
   b. g(f(x)) = x
2. Step 2 – if both compositions equal x, the 2 functions are Inverses
3. Step 3 – if both compositions do not equal x, the 2 functions are not Inverses

Determine whether each pair of functions are inverse functions <u>using composite functions</u>.

9) $f(x) = 3x - 6$

$g(x) = \frac{x+6}{3}$

$f(g(x)) = 3\left(\frac{x+6}{3}\right) - 6$

$= x + 6 - 6$

$= x$

$g(f(x)) = \frac{(3x-6)+6}{3}$

$= \frac{3x}{3}$

$= x$

yes f(x) & g(x) are inverses of each other

10) $f(x) = 4x^2 - 3$

$g(x) = \frac{\sqrt{x+3}}{2}$

yes f(x) & g(x) are inverses

$f(g(x)) = 4\left(\frac{\sqrt{x+3}}{2}\right)^2 - 3$

$= 4\left(\frac{x+3}{4}\right) - 3$

$= x + 3 - 3 = x$

$g(f(x)) = \frac{\sqrt{(4x^2-3)+3}}{2}$

$= \frac{\sqrt{4x^2}}{2} = \frac{\sqrt{2 \cdot 2 \cdot x \cdot x}}{2} = \frac{2x}{2} = x$

11) $f(x) = \log_2 x$

$g(x) = 2^x$

$f(g(x)) = \log_2(2^x) = x$

$g(f(x)) = 2^{(\log_2 x)} = x$

Yes f(x) & g(x) are inverses

12) $f(x) = \log_3(x) + 1$

$g(x) = 3^{x-1}$

f(x) & g(x) are yes inverses

$f(g(x)) = \log_3(3^{x-1}) + 1$

$x - 1 + 1 = x$

$g(f(x)) = 3^{(\log_3(x)+1} - 1$

$= 3^{\log_3(x)+1-1}$

$= x$

13) $f(x) = \log_4(x) - 2$

$g(x) = 4^x$

$f(g(x)) = \log_4(4^x) - 2$

$= x - 2$

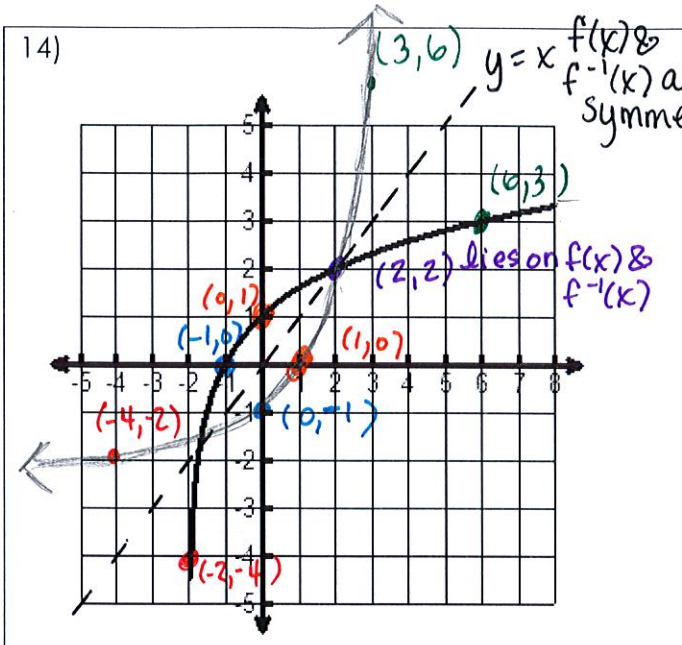
No f(x) & g(x) are not inverses.

Inverses graphically – How do we find the Inverse of a function graphically?

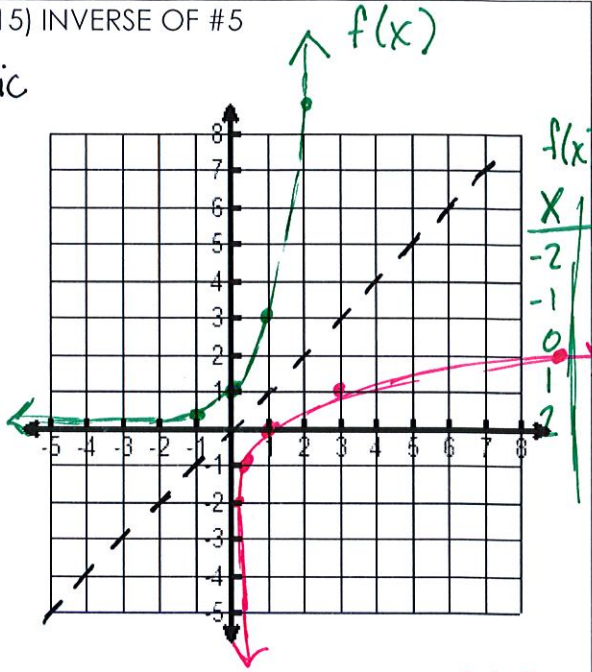
1. Step 1 – plot 4 to 5 points on the given graph
2. Step 2 – write down each ordered pair that corresponds with each plotted point
3. Step 3 – Switch the x values and y values in the ordered pairs
4. Step 4 – plot the NEW points on the same graph
5. Step 5 – play connect the dots to see the graph of the inverse

Find the inverse of each function.

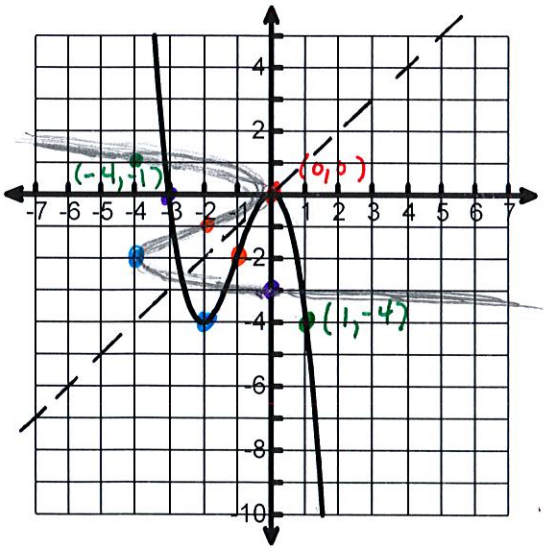
14)



15) INVERSE OF #5



16)



17) INVERSE OF # 6 $f(x) = 2^{x-1}$